Journal of Hazardous Materials, 26 (1991) 187-202
Elsevier Science Publishers B.V., Amsterdam

Small spills of heavy gas from continuous sources

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(Received February 1, 1990; accepted July 17, 1990)

Abstract

Constant-rate releases of argon in a channel of constant width as well as in a thirty-degree sector, have been used to model the spreading of ethylene oxide from an evaporating pool. Profiles of velocity and concentration have been measured at various distances from the source. A theoretical model, valid in cases for which the flow is terminated by a free overfall, has been utilized to determine the entrainment rate and energy loss. This simple model can be used also for predictive purposes when empirical relations for entrainment and dissipative losses are prescribed.

1. Introduction

Existing methods for predicting the spreading of heavy gas plumes, are better suited to the large-scale spectacular spills that only rarely occur, than to the many small leaks and spills that occur in routine operations in the process industries. Gas flow rates of the order of ten liters per second appear trivial in comparison with those postulated for catastrophic spills, but when the gas is highly toxic or flammable, even a small release inside a laboratory hall or in a sheltered work area represents a real and present danger. Little is known about the fate of such small spills, their rate of spreading and dilution and how long the heavy gas remains in the sublayer.

The present study deals with ethylene oxide, a fluid which is used in large quantities in the chemical industry. It is toxic, explosive, carcinogenic and apparently indispensable. In the case of interest, the normal transfer operation from railroad tank car to plant storage tank may result in a ground spill of about 5 kg liquid (the content of the transfer-hose). At temperatures in excess of 10.5°C, the liquid would evaporate and produce an invisible low plume of unknown extent and concentration. Our first attempt at prediction based on the usual box-model was unsuccessful for the lack of relevant data, but also as we will see, due to uncertainty about the distant boundary condition.

A review of environmental conditions at the site, indicates that a ground temperature of 20°C and no air movement, would be a suitable "worst case". (In a complete risk analysis, cases involving non-negligible wind speed, could

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also be of interest. The methods of analysis are discussed by Britter [1].) Under these conditions, the liquid spill would form a pool of about three square meters, and its content would evaporate in 9–10 minutes giving a gas flow rate of about 3.5–4.5 liters per second. (The pool area was determined by a series of simulated spills, using water rather than ethylene oxide to fill the transfer-hose. The evaporation rate was determined from a heat balance. The rate of heat transfer from the ground to the pool, was calculated by means of the unsteady one-dimensional heat-conduction equation using the properties of asphalt. The numerical results obtained served to design the experiment, but are otherwise not particularly important). A cursory box-model analysis indicates a front speed of about 0.1 m/s and a range of 40–60 m in the time the source remains active. Such an analysis seems deficient for several reasons. At a distance of about 10^3 times the plume height, the front condition is not likely to control the flow near the source. But on dropping this condition the problem becomes indeterminate, we can not predict the height and outflow velocity at the source which is assumed to have negligible initial momentum; only the product of velocity and height would be known from continuity.

We will here propose an alternate approach, applicable to continuous sources, based on a different downstream condition.

2. Gas plumes from continuous sources

The box-model solution for a steady point source, $r(t)$, started at $t=0$, can be readily evaluated in terms of the source strength $\dot{m}$, i.e.

$$r(t)=\left[\frac{k\Delta\rho g}{\rho_a}\frac{16}{9\pi}\frac{\dot{m}}{\rho_g}\right]^{1/4} t^{3/4} \tag{1}$$

where k is an empirical constant, $\Delta\rho$ equals $\rho_g-\rho_a$ which is the difference between the gas and air density, and t denotes the time, while g is the gravitational constant (see for instance Jacobsen and Fanneløp, [2]). In a recent review Britter [1] confirms that a solution of this form is consistent with his experimental information, but he was unable to find a true similarity solution based on the shallow-layer equations. Grundy and Rottmann [3] concluded that there is no similarity solution for this problem for $\dot{m}\sim t^{\alpha}$, $\alpha\neq 0$. But Waldman et al. [4] did derive similarity solutions for the analogous problem of the spreading of oil slicks from time-varying sources. Some of these solutions were derived independently later by Huppert [5], and certain experiments with stepwise changes in the source strength were conducted by Maxworthy [6]. Although interesting, the problem of the possible non-existence of similarity solution for certain source conditions is not likely to be of prime concern in our case. The solution obtained, based on the usual similarity conditions, will not be physically realistic for very large times. Viscous effects, energy losses and interactions with distant boundaries and obstacles will have to be considered to obtain plausible results.

At the actual spill site, where the gas is free to spread only in a limited sector, the spreading surface is intersected at some distance by a drainage channel. To predict the plume velocity and height upstream, we have postulated that the gas flows into this channel at the critical velocity in analogy with a hydraulic overfall. A critical-flow condition produced by a submerged barrier (weir), by a downhill slope or similar flow configurations known in hydraulics, is believed to represent the type of boundary condition (or downstream control) required in the analysis of heavy-gas plumes from continuous sources. To test the hypothesis of a critical flow for the case of interest, we have built a small channel, about one-to-one in scale, where the heavy gas flow after a run of about ten source diameters, is terminated by an overfall. We have measured the velocity both in the plume and at the overall to check our assumptions. We have also measured the concentration and height and thereby determined the rate of entrainment. We have attempted to check the accuracy and validity of the results by a numerical simulation of the spreading of the heavy gas plume, and by estimates of dissipative losses, with reasonable success. As velocity measurements in the range 0–0.2 m/s are very difficult with conventional means, the accuracy of these measurements represents the limiting factor in the mass and flow balances. In addition to the flow measurements, we also made use of visualization. In spite of the low Reynolds number and the absence of thermal effects, flow structures in the form of longitudinal streaks or rolls are found. We shall discuss the experimental setup and the results in detail in a later section.

3. Theoretical considerations

In the case of a sudden release, the "distant" boundary condition which determines the front position, r_f, and spread, u_f, is known to be

$$u_f=\frac{dr_f}{dt}=[kg'h]^{1/2} \tag{2}$$

where

$$g'=g\frac{(\rho-\rho_a)}{\rho_a}$$

and h is the plume's height. This relation represents the rate of advance of a density current in the absence of dissipative processes. Equation (2) cannot be used when the front approaches an overfall or a distant obstacle.

In the steady state we can replace eqn. (2) by a condition which specifies "critical flow" at an appropriate location. The relevant wave speed u_c can be determined from the shallow-layer equations, as in Jacobsen and Fanneløp, [7].

$$u_c = [\bar{g} h_c]^{1/2} \tag{3}$$

where

$$\bar{g} = g \frac{(\rho - \rho_a)}{\rho_g}$$

u_c and h_c are the critical velocity and the critical height respectively which we expect to find at an overall. (Note that $g' \approx \bar{g}$ only when $\rho_a/\rho_g \simeq 1$). The volume flow rate of gas, $\dot{q}_g$, is given by

$$\dot{q}_g = \frac{\dot{m}}{\rho_g} \tag{4}$$

is considered known, and for steady and quasi-steady sources continuity requires

$$\dot{q}_g = (2\pi r)^j u h_g \quad \begin{Bmatrix} j=0 \text{ planar flow} \\ j=1 \text{ radial flow} \end{Bmatrix} \tag{5}$$

if h_g is the fictitious plume height of pure gas and top-hat profiles are used. Due to entrainment the real height will be higher. The critical speed is determined by the excess hydrostatic head and should be unaffected by entrainment. It follows:

$$u_{cg} = [\bar{g}_g h_{cg}]^{1/2} = \left[\frac{\rho_g - \rho_a}{\rho_g} g h_{cg} \right]^{1/2} \tag{6}$$

Continuity and critical flow combined give the conditions

$$u_{cg} = \left[\frac{\bar{g}_g \dot{q}_g}{(2\pi R)^j} \right]^{1/3}, \quad h_{cg} = \left[\frac{\dot{q}_g^2}{\bar{g}_g (2\pi R)^{2j}} \right]^{1/3} \tag{7 a,b}$$

where index g indicates pure gas and R is the radial distance to the overfall.

The total volume flow rate can be expressed $\dot{q} = \dot{q}_a + \dot{q}_g$ where $\dot{q}_a$ is the air flow rate due to entrainment. As $\dot{q}_g$ is constant we write

$$\frac{\mathrm{d}}{\mathrm{d}r}(r^j u h_a) = r^j v_e \tag{8}$$

where v_e is the entrainment velocity and $h_a = h - h_g$ where h is the total plume height. We assume, in accord with our experimental observations, that u can be expressed approximately as a constant fraction of u_c. We write correspondingly the entrainment velocity v_e as $v_e = \beta u = \beta_c u_c$ where $\beta_c = (u/u_c)\beta$. A first estimate of $\dot{q}_a$ is obtained by integrating eqn. (8)

$$\dot{q}_{\mathrm{a}}(r)=(2\pi r)^j u h_{\mathrm{a}}=\frac{(2\pi)^j}{j+1}r^{j+1}\beta_{\mathrm{c}}u_{\mathrm{c}} \tag{9a}$$

The concentration at $r=R$, $\bar{c}=\dot{q}_{\mathrm{g}}/(\dot{q}_{\mathrm{g}}+\dot{q}_{\mathrm{a}})$ follows:

$$\bar{c}=\frac{(j+1)h_{\mathrm{cg}}}{\beta_{\mathrm{c}}R+(j+1)h_{\mathrm{cg}}} \tag{9b}$$

This estimate can be improved as the air entrainment leads to a momentum drag which can be offset only by a corresponding variation in the height, i.e. $\mathrm{d}h_{\mathrm{g}}/\mathrm{d}r<0$. But for the purpose of obtaining an improved estimate, it will be better to consider the equations of motion rather than ad hoc approximations.

The momentum and the continuity equations for the steady case can be obtained from the full equations [7,14] or derived directly from momentum and mass balances (top-hat profiles) using the hydrostatic pressure relation. The relevant shallow layer equations are given below.
Conservation of mass:

$$\frac{\partial}{\partial r}(hu)+jh\frac{u}{r}=v_{\mathrm{e}} \tag{10}$$

Conservation of horizontal plume momentum:

$$\frac{\partial}{\partial r}(\rho h u^2)+j\rho h\frac{u^2}{r}+\frac{\partial}{\partial r}\left[\frac{g}{2}(\rho-\rho_{\mathrm{a}})h^2\right]=-\tau_{\mathrm{w}}\rho_{\mathrm{a}} \tag{11}$$

We note that the entrainment drag is accounted for in this formulation [7]. Included is also τ_{w}, the viscous stress at the wall. Apart from the region near the overfall, where a rapid acceleration takes place, the change in dynamic head will be small. The effects of the wall shear is found also to be small. By assuming a laminar boundary layer value (Blasius) for τ_{w}, the numerical integration of (10) and (11) give about the same result as in the case $\tau_{\mathrm{w}}=0$. (Actual difference, about one percent.) Accurate estimates of τ_{w} are therefore not required.

The low Reynolds number of our experiment $\mathrm{Re}=\rho_{\mathrm{g}}uR/\mu_{\mathrm{g}}\approx 10^4$, indicates a laminar boundary layer, but the presence of longitudinal streaks gives evidence of a more complex flow. A solution to the spreading problem, can in principle be obtained by integrating eqns. (10) and (11), for the known initial flow rate and subject to a specified entrainment relation (8). Although the flow rate (or uh) is known initially, the actual height and velocity at the start of integration $L=0$, will be determined by the flow conditions far downstream, i.e. at the free overfall. To establish this boundary condition, we should take into account both the upstream influence and the energy loss associated with the free overfall. Our flow problem is analogous to a classical problem in open channel

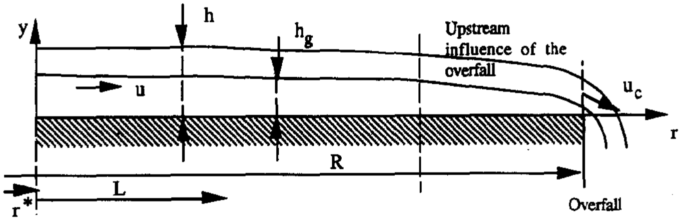

Fig. 1. Sketch of flow configuration with the nomenclature used.

hydraulics and useful information (albeit only for unstratified flows) can be found in text books, e.g. Henderson [8] indicates an upstream influence of about 3 to 4 times the critical-layer depth, i.e. 3 to 4 times h_{cg} from (7b). Our measurements in stratified heavy-gas flow, indicate values an order of magnitude greater.

We have made use of the measured values in the integration of eqns. (10) and (11) in two ways. By successive iteration we have found values of v_e (or β) which, together with the equations of motion produce a variation in argon concentration with r which agrees with the experiments. This determines the entrainment rate β. The integration is terminated at the point where the upstream influence of the overfall is first noted (see Fig. 1). The calculated height at this location is required to match the experimental value. This determines the energy loss associated with the overfall. This loss is found to agree well with that of an broad-crested weir, as estimated by Knapp [9]. Expressed in terms of the pure argon plume, the loss in energy amounts to $\Delta h_v = 0.46\, u_c^2/2g$ or equivalently a loss in height $\Delta h = 0.732\, h_c$. The extent of upstream influence depends on the configuration used, and will be discussed in a later section.

4. Experimental setup

In view of the risks associated with ethylene oxide, we have used argon for the experiments. The density and diffusion characteristics for the two gases are closely matched. To simulate radial spreading, the gas was released at the apex of a 30-degree sector (see sketch). The gas was allowed to flow vertically into the release area (simulating the evaporating pool) through a bed of polystyrene balls. The initial flow had no lateral momentum and very low velocity in the vertical direction. The geometry of the channel is indicated in the sketch (Fig. 2). The sector is terminated by a straight cut about 4.32 m (measured along the centerline) from the edge of the simulated pool. The flow in the entrance and overfall regions is therefore not radial, but the deviations are small.

The rapid thinning of the plume in the 30° sector, makes the measurements (in particular of the velocity) more difficult. To obtain results which were easier to measure, for comparison with the theoretical assumptions, we inves-

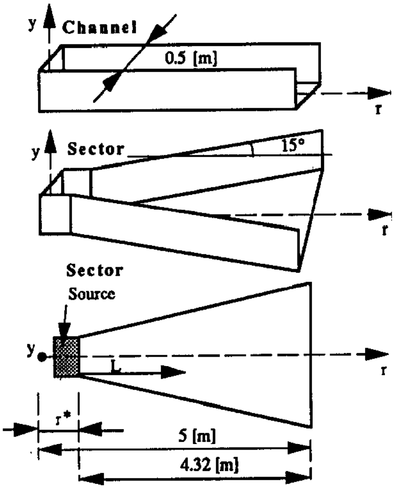

Fig. 2. Geometry of channel and 30° sector.

tigated also the flow in a straight channel using the same gas-release arrangement (see Fig. 2). The channel and the sector spreading surfaces were raised from the floor to give a height of fall of about one half meter. The spreading area was protected by rather high sidewalls, and the experiment was further shielded from air movements in the laboratory hall by mobile wall segments.

The flow was studied by means of smoke visualization, by concentration measurements and by pulsed-wire velocity measurements. The visualization provided qualitative information and photographic records, and displayed clearly the longitudinal streaks visible only in the 30° sector. The concentration measurements made use of a commercial measurement device, which depends on the paramagnetic properties of oxygen. In measures in effect the oxygen deficiency relative to air. It is a suction device, in our case equipped with a narrow radial suction slit in the horizontal plane. Based on the size of the device and its suction rate, it is estimated that the measured concentration is an average over a volume of 6 mm diameter. This tends to smear out the gradients, in particular at the edge of the plume. The data presented represent long-time averages, and the presence of longitudinal rolls could not be detected from the measurements. The pulsed hot wire anemometer, was originally built by Krogstad [10], and could measure the velocities with reasonable accuracy down to about 0.07–0.10 m/s. With this limitation only the velocity in the central part of the plume could be determined. Lack of complete velocity profiles made it difficult to establish acceptable mass and volume balances. We consider the results based on concentration measurements (and deduced heights) as more reliable than those based on velocity.

5. Results and discussion

For the visualization cold smoke was injected into the release area a little prior to and during a run. The smoke initially present, accumulated in and moved with the front wave as seen in Fig. 3. Figure 3(a) shows the rear view of the complex front structure as revealed by a laser sheet; Fig. 3(b) shows the frontal view of the same phenomenon, this time in normal light. The longitudinal streaks visible in Fig. 3(b) behind the front were present also in the

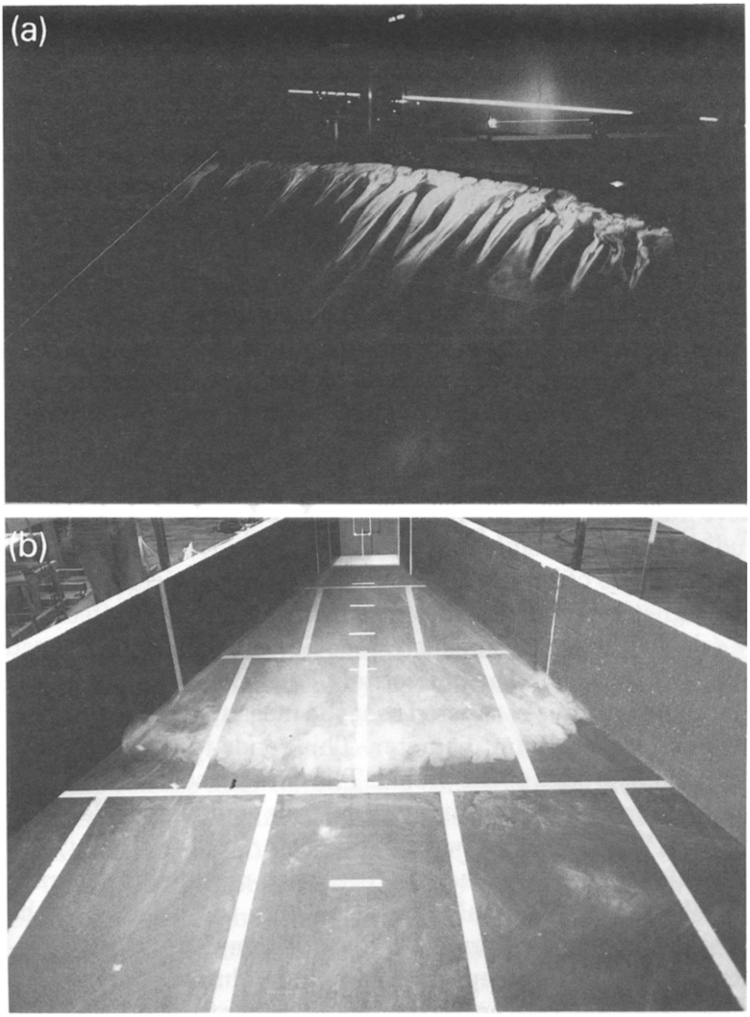

Fig. 3. (a) Rear view of front wave in sector as revealed by smoke when illuminated by a laser sheet. (b) Upstream view of flow structures in sector as revealed by smoke in normal light.

steady flow established after the front wave disappeared over the overfall. These longitudinal structures in the steady flow, were observed only in the sector and not in the channel. The entrainment process could be influenced by what appears to be longitudinal rolls, but further details could not be established with the instrumentation available.

Figures 4 and 5 show the results of the concentration and the velocity measurements in the channel. The concentration at the first station adjacent to the source region reaches 100% argon, and the peak concentration is still as high as 50% just upstream of the overfall. What is striking is that the plume height remains nearly constant. The fall-off in the height required to balance the friction and inertia terms is equalized by the rate of entrainment so that the plume height remains about the same. The peak velocity is seen to vary in a narrow band as well, between 0.07 m/s and 0.12 m/s outside the overfall area. The velocity measurements indicate an influence of the overfall already at $L=3.5$ m or 0.8 m upstream, whereas the concentration measurements show no apperent effect at $L=4.0$ m. (We consider the concentration measurements as more reliable.) The "average" velocity at the overfall is ca. 0.26–0.28 m/s vs. a calculated critical value 0.15 m/s for the pure argon flow. These values may not be comparable as the critical state, in the hydraulic analogy, appears somewhat upstream of the edge.

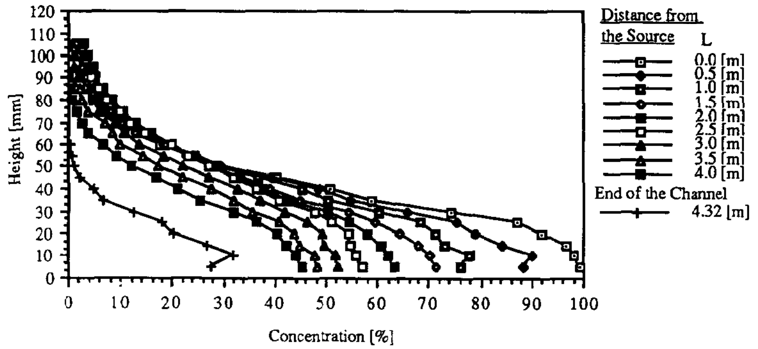

Fig. 4. Concentration of argon in the channel; steady state.

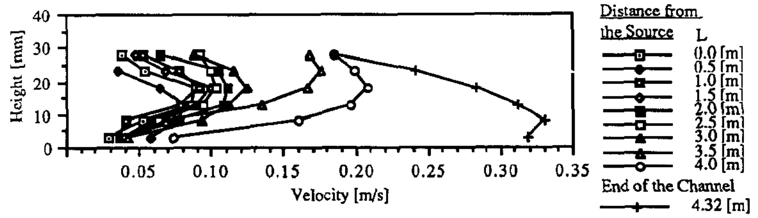

Fig. 5. Velocity of the heavy gas in the channel; steady state.

The results of the concentration and velocity measurements in the sector are shown in Figs. 6 and 7. The concentration falls more rapidly with distance relative to the channel flow, but the height remains also in this case about constant outside the overfall region. The height is considerable smaller than in the channel due to the radial spreading. The peak velocities are nearly the same, they fall in the range 0.09–0.11 m/s. The averaged velocity at the overfall is comparable in magnitude to the estimated critical value, $u_{cg}=0.11$ m/s, for the pure argon plume. It is seen from Figs. 6 and 7 that the upstream influence is rather weak for the flow in the sector.

In order to deduce (through iteration) the relevant entrainment rates by means of eqns. (8), (10) and (11), it is necessary to convert the measured air–gas plume to an equivalent pure argon plume, using the measured concentration profiles. The heights of the pure argon plume, for the sector and channel respectively, are shown in Figs. 8(a) and 9(a). In these data, the fall in the height required to balance the momentum drag of the entrained air, is clearly displayed. As the total argon flow rate is given, the velocity of the pure gas plume can also be established, and the results are shown in Fig. 8(b) and 9(b). To establish the entrainment rates, eqns. (8), (10) and (11) have been integrated subject to the known initial conditions as well as the following; (a) The final height should agree with the measured value just upstream of the overfall; and (b) The slope dh_g/dr along the sector or channel should agree with the

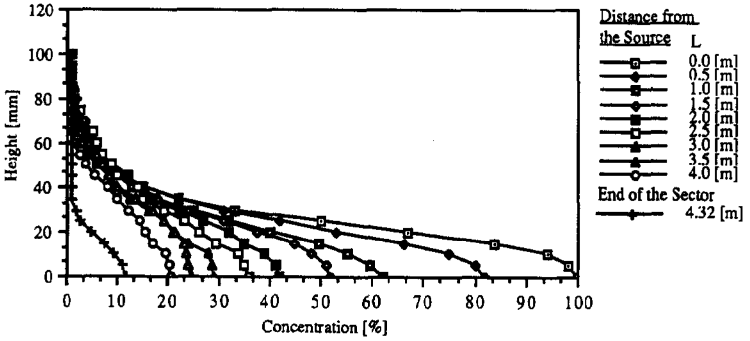

Fig. 6. Concentration of argon in the 30° sector; steady state.

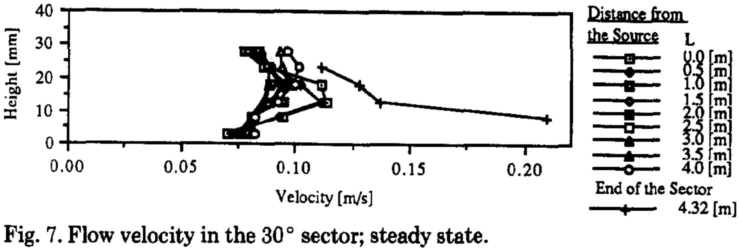

Fig. 7. Flow velocity in the 30° sector; steady state.

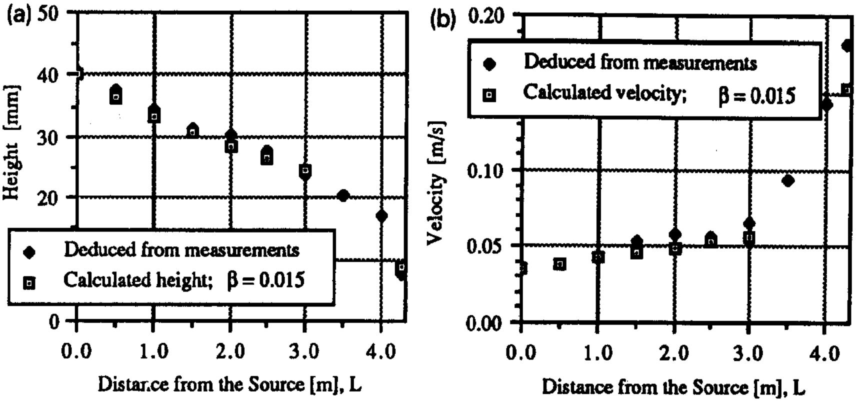

Fig. 8. (a) Calculated and measured height in the channel of a plume that contains 100% heavy gas.
(b) Calculated and measured velocity in the channel of a plume that contains 100% heavy gas.

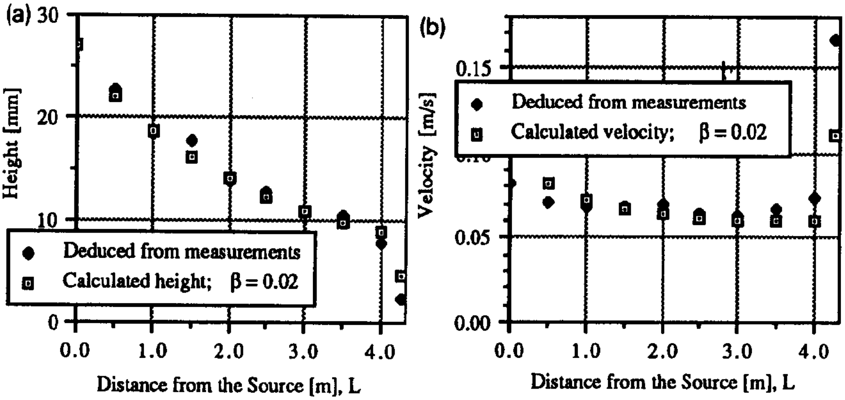

Fig. 9. (a) Calculated and measured height in the sector of a plume that contains 100% heavy gas.
(b) Calculated and measured velocity in the sector of a plume that contains 100% heavy gas.

measured values. This two-point boundary-value problem was solved by successive iterations using assumed values of v_e in (8). The results are indicated in Figs. 8(a) and 9(a). The best match in the case of the channel was obtained with an entrainment corresponding to $\beta = 0.015$. For the sector the corresponding value was $\beta = 0.02$. As a second check on the validity of the results, we can compare the theoretical and experimental velocities. As shown in Figs. 8(b) and 9(b), the agreement is acceptable except in the region nearest to the overfall.

Figure 10 shows the calculated concentration also obtained through the in-

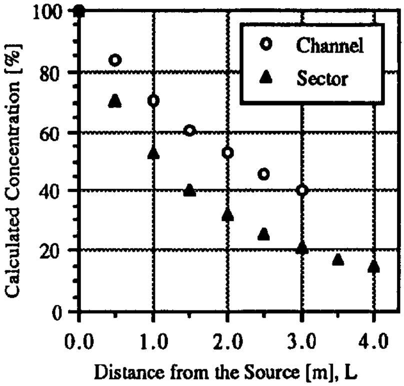

Fig. 10. Calculated concentration.

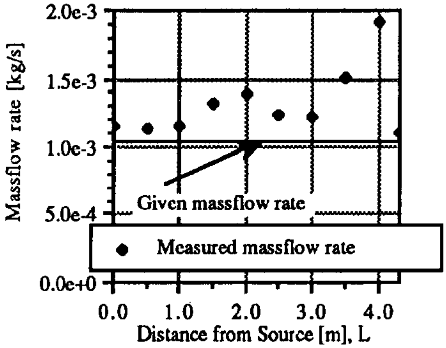

Fig. 11. Mass flow in the channel.

tegration of eqns. (8), (10) and (11). These results are to be compared directly with those obtained from the simple theory proposed herein, i.e. eqn. (9b).

Figures 11 and 12 show the attempts to balance mass in the channel and in the sector respectively, based on the measured concentrations and velocities.

The weak link here is the velocity measurements, as data have been obtained only in the central part of the plume. The profiles have been "completed" assuming similarity between the concentration and the velocity profiles in the free shear layer, and "no slip" at the wall. The discrepancies can be attributed primarily to the measured velocities in the lower range which have poor accuracy. The deduced entrainment parameters are $\beta = 0.01$ (Channel, $Ri = 20$) and $\beta \approx 0.022$ (sector, $Ri = 10$), and confirm at least the order of magnitude of those found from the concentration measurements. The Richardson numbers indicated here and in Fig. 3, are defined by $Ri = g'h/u^2$ and have been evaluated at $R = 4.0$ m.

Based on the experimental results we can now check on the simple theory, proposed herein, to see whether it gives reasonable results. We consider first

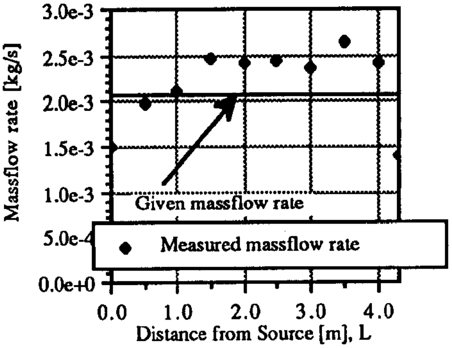

Fig. 12. Mass flow in the sector.

the channel, where we have a mass flow rate $\dot{m}_g=0.0011$ kg/s, and critical velocity and height $u_{cg}=0.15$ m/s and $h_{cg}=0.009$ m. The averaged ratio $u_g/u_{cg}\approx 0.33$ and $\beta_c=0.33\beta\approx 0.005$. On inserting these values into (9b), we can calculate the concentration as a function of distance r. At the final station considered when integrating eqns. (8), (10) and (11), we have (Figs. 8a,b and 10) $L=3$ m, $R\approx 3.27$ m and $\bar{c}=0.36$, which is in reasonable agreement with the result shown in Fig. 10. The last station measured upstream of the overfall is, for the channel, $L=4.0$ m or $R=4.27$ m. The value obtained from eqn. (9b) is here $\bar{c}=30\%$. The averaged value from the measurements shown in Fig. 4 is about 26% at the same station.

On repeating the prediction for the sector we have the following data: $\dot{m}_g=0.00215$ kg/s, $u_{cg}=0.11$ m/s, $h_{cg}=0.0045$ m, $\alpha\approx 0.55$ and $\beta_c\approx 0.011$. This gives ($R=4.93$ m, $j=1$) $\bar{c}=0.14$ or 14%. This is quite close to the value shown in Fig. 10. The averaged value from the last measured concentration profile in the sector is about 12%, somewhat below the predicted value. The non-radial flow in the source region and near the overfall could have an influence on this result.

6. Conclusions

More than a hundred models have been proposed to date for the prediction of plume development and concentration resulting from unsteady (instantaneous) releases of heavy gas. For steady releases in the absence of wind, an acceptable model has yet to be proposed.

The frontal (intrusion) condition which makes possible the simple similarity (box-model) solution for unsteady releases, becomes irrelevant in the steady case and new downstream boundary conditions must be specified, as pointed out by Britter [1]. Relevant boundary conditions can be established by analogy with flows in open-channel hydraulics, such as hydraulic jumps associated with blockage [1] or critical flow over weirs and free overfalls. With reference to the latter, we have proposed herein a simple theory which assumes critical flow velocity of the air–gas mixture at an overfall. These conditions together with the known source strength and flow geometry, as well as a specified (empirical) entrainment parameter, allows a complete solution of reasonable accuracy to be found with little computational effort.

To check on this method, a series of experiments have been conducted in radial and straight channel flows, both terminated by a free overfall. The gas concentration and the flow velocity have been measured for both flow geometries, and in addition, flow visualization has been employed to investigate special flow features (longitudinal streaks). We have found that the stratified heavy-gas flow near the overfall is quite similar in character to the analogous case in open channel hydraulics; the loss in flow energy is of comparable magnitude but the upstream influence reaches much further in the stratified case.

To deduce the effective entrainment velocity or coefficient from the data, the full equations of motion for the gas/air mixture have been integrated as a two-point boundary-value problem subject to the known initial source strength at the upstream end, and forced to agree with the measured plume height at the downstream end. The latter was evaluated just upstream of the region of influence of the overfall. The effective entrainment rate was found by successive iterations by matching the measured and calculated shapes of the equivalent plumes of pure gas along the channel. As a further check on the consistency of the results obtained, the velocities determined from the measured (pure gas) plume height and the known gas-flow rate on one hand, and the velocities calculated from the equations of motion on the other, were found to agree well for the entrainment rates determined through successive iterations. For the channel we found $\beta = 0.015$ ($Ri \approx 20$) and for the sector $\beta = 0.02$ ($Ri \approx 10$). It is possible that the higher value in the sector in part is attributable to the presence of longitudinal rolls.

The results of the simplified theory is found to be in good agreement with that obtained by integrating the full equations of motion assuming top-hat profiles (see Fig. 10). A comparison between the simplified theory and the experiments requires the experimental profiles to be averaged over the observed plume height. The averaged values are 10–15% below the predictions, an acceptable agreement. It appears that the simple theory proposed, based on "critical flow" downstream conditions offers a practical alternative of reasonable accuracy for predicting plume concentrations as compared to a full integration of the equations of motion or to the use of experiments (e.g."physical models"), as long as a reasonable estimate of the entrainment rate can be obtained.

For steady flows of cold evaporated nitrogen over a dry surface, Ruff et al. [11] reported $\beta \approx 0.0028$–0.0034. For cold nitrogen over water, Colenbrander and Puttock [12] found values about 25% higher whereas Poag [13] measured entrainment rates of the order of 0.01 over water for an initially dry nitrogen plume. The available experiments with evaporated cryogenic gases flowing steadily in a channel give consistently lower entrainment rates than ours, but loss of (negative) buoyancy through mass and heat transfer, is likely to play an important role.

Acknowledgement

The present study was initially organized as an exchange student thesis project for Mrs. Hilary Khoo (now graduate student, Imperial College) who first pointed out the inadequacy of existing heavy-gas models for this type of release. Dipl. ing. Guido Sutter assembled the original experimental setup. The problem was suggested by Sandoz AG who also donated the necessary equipment for the first series of measurements.

Notation

β	Entrainment factor
g	Acceleration due to gravity
h	Height of the plume
h_a	Height associated with entrainment, $h_a = h - h_g$
h_c	Height of the plume at critical flow velocity
h_g	Height of a plume with 100% gas
k	Empirical constant in eqn. (1)
L	Length from source region
$\dot{m}$	Mass flow rate
ρ	Density of the gas–air mixture
ρ_a	Density of air
ρ_g	Density of gas
$\dot{q}$	Volume flow rate
r	Coordinate in the sector and channel
r_f	Coordinate of the front of the plume
R	Distance from source to overfall
Re	Reynolds number
Ri	Richardson number
τ_w	Viscous stress at the wall
t	Time
u	Velocity in direction r
u_f	Front velocity
u_c	Critical velocity defined by eqn. (3)
v_e	Entrainment velocity defined by eqns. (8) and (9)
μ_g	Viscosity of gas

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